BT-MRE-I & II(S)-05.15-0567 Reg. No. [| | | | | | | |]

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B. Tech. Degree I &II Semester Supplementary Examination in Marine Engineering May 2015

MRE 101 ENGINEERING MATHEMATICS I

Time: 3 Hours Maximum Marks: 100

(5× 20 = 100)

I. (a) Verify Legrange's Mean Value Theorem for log × in the interval $[1,e]$. (6)

(b) Evaluate $\lim_{x\to 0} \sin x^{\tan x}$. (7)

(c) Evaluate $\lim_{x\to\pi/4} \dfrac{\sec^2 x - 2\tan x}{1+\cos 4x}$ (7)

OR

II. (a) Find the asymptotes of $x^3 - x^2y - x - 6 = 0$. (6)

(b) Find the n^{th} derivative of $\log(1+x)$. (6)

(c) If $y = \sin(m \sin^{-1} x)$ prove that (8)

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

III. (a) Verify Euler's theorem for the function $U = \dfrac{x(x^3 - y^3)}{x^3 + y^3}$. (7)

(b) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$. Find $\dfrac{du}{dt}$ as a total derivative. (7)

(c) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. (6)

OR

IV. (a) Prove that the sphere is the solid figure of revolution which for a given surface area, has maximum volume. (10)

(b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for the construction. (10)

V. (a) Find the equation of the tangent at the $pt\,(x_1, y_1)$ on the parabola $y^2 = 4ax$. (7)

(b) Prove that 3 normals can be drawn to a parabola from any *pt* in its plane and that the sum of the ordinates of the feet of the normals is zero. (7)

(c) Find the condition for $lx + my + n = 0$ to be a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (6)

OR

VI. (a) Find the equation of the asymptotes of the hyperbola $2x^2 + 2xy - 3x + y = 0$. (10)

(b) Prove that the orthocenter of any triangle inscribed in a rectangular hyperbola lies on the curve. (10)

VII. (a) Find a reduction formula for $\int e^{ax} \cos^n x \, dx$. Hence evaluate $\int_0^{\pi/2} e^{2x} \cos^3 x \, dx$. (10)

(b) Find the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$. (10)

OR

VIII. (a) Evaluate the integral by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dxdy$. (7)

(b) Find the area lying between the parabola $= 4x - x^2$ and the line $y = x$. (6)

(c) Find the volume generated by the revolution of the cardiod $r = a(1 - \cos\theta)$ about its axis. (7)

IX. (a) Prove that $\left[\overline{B}X\overline{C}\overline{C}X\overline{A}\overline{A}X\overline{B}\right] = \left[\overline{A}\overline{B}\ \overline{C}\right]^2$ (10)

(b) A particle moves along the curve $R = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$ where t denotes time. Find the magnitudes of acceleration along the tangent and normal at time $t = 2$. (10)

OR

X. (a) Show that the vector field defined by $F = (x^2 + xy^2)i + (y^2 + x^2y)j$ is conservative and find the scalar potential. Hence evaluate $\int F.dr$ from $(0,1)$ to $(1,2)$. (10)

(b) Prove that (10)

(i) curlcurl F=grad div F- ∇^2 F

(ii) curl grad F=0

(iii) div grad F= ∇^2 F

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